

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Mechanics 3

MM03

Wednesday 21 June 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The time T taken for a simple pendulum to make a single small oscillation is thought to depend only on its length l , its mass m and the acceleration due to gravity g .

By using dimensional analysis:

- (a) show that T does **not** depend on m ; (3 marks)
- (b) express T in terms of l , g and k , where k is a dimensionless constant. (4 marks)

- 2 Three smooth spheres A , B and C of equal radii and masses m , m and $2m$ respectively lie at rest on a smooth horizontal table. The centres of the spheres lie in a straight line with B between A and C . The coefficient of restitution between any two spheres is e .

The sphere A is projected directly towards B with speed u and collides with B .

- (a) Find, in terms of u and e , the speed of B immediately after the impact between A and B . (5 marks)
- (b) The sphere B subsequently collides with C . The speed of C immediately after this collision is $\frac{3}{8}u$. Find the value of e . (7 marks)

- 3 A ball of mass 0.45 kg is travelling horizontally with speed 15 m s^{-1} when it strikes a fixed vertical bat directly and rebounds from it. The ball stays in contact with the bat for 0.1 seconds.

At time t seconds after first coming into contact with the bat, the force exerted on the ball by the bat is $1.4 \times 10^5(t^2 - 10t^3)$ newtons, where $0 \leq t \leq 0.1$.

In this simple model, ignore the weight of the ball and model the ball as a particle.

- (a) Show that the magnitude of the impulse exerted by the bat on the ball is 11.7 N s , correct to three significant figures. (4 marks)
- (b) Find, to two significant figures, the speed of the ball immediately after the impact. (4 marks)
- (c) Give a reason why the speed of the ball immediately after the impact is different from the speed of the ball immediately before the impact. (1 mark)

- 4 The unit vectors \mathbf{i} and \mathbf{j} are directed due east and due north respectively.

Two cyclists, Aazar and Ben, are cycling on straight horizontal roads with constant velocities of $(6\mathbf{i} + 12\mathbf{j}) \text{ km h}^{-1}$ and $(12\mathbf{i} - 8\mathbf{j}) \text{ km h}^{-1}$ respectively. Initially, Aazar and Ben have position vectors $(5\mathbf{i} - \mathbf{j}) \text{ km}$ and $(18\mathbf{i} + 5\mathbf{j}) \text{ km}$ respectively, relative to a fixed origin.

- (a) Find, as a vector in terms of \mathbf{i} and \mathbf{j} , the velocity of Ben relative to Aazar. (2 marks)
- (b) The position vector of Ben relative to Aazar at time t hours after they start is $\mathbf{r} \text{ km}$.

Show that

$$\mathbf{r} = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j} \quad (4 \text{ marks})$$

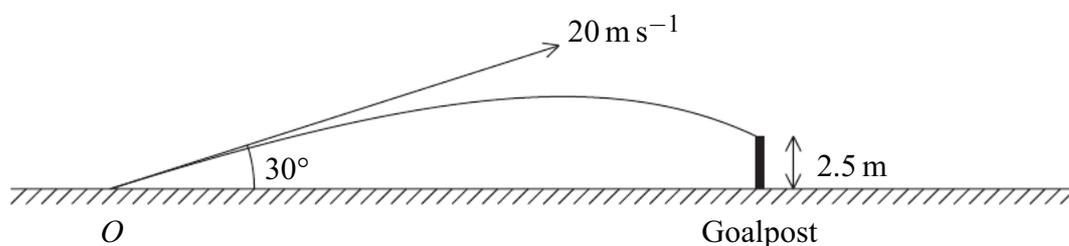
- (c) Find the value of t when Aazar and Ben are closest together. (6 marks)
- (d) Find the closest distance between Aazar and Ben. (2 marks)

- 5 A football is kicked from a point O on a horizontal football ground with a velocity of 20 m s^{-1} at an angle of elevation of 30° . During the motion, the horizontal and upward vertical displacements of the football from O are x metres and y metres respectively.

- (a) Show that x and y satisfy the equation

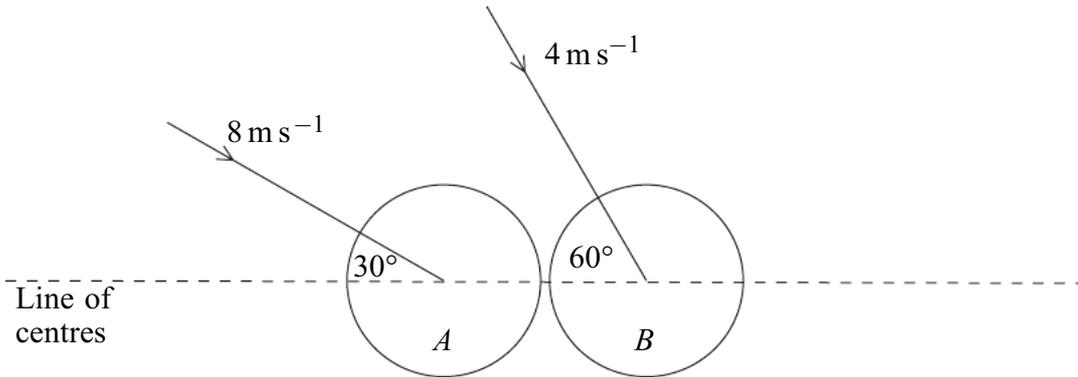
$$y = x \tan 30^\circ - \frac{gx^2}{800 \cos^2 30^\circ} \quad (6 \text{ marks})$$

- (b) On its downward flight the ball hits the horizontal crossbar of the goal at a point which is 2.5 m above the ground. Using the equation given in part (a), find the horizontal distance from O to the goal. (4 marks)



- (c) State **two** modelling assumptions that you have made. (2 marks)

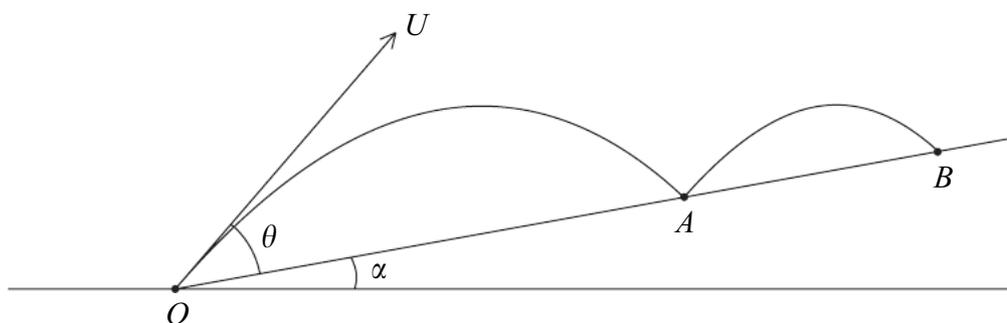
- 6 Two smooth billiard balls A and B , of identical size and equal mass, move towards each other on a horizontal surface and collide. Just before the collision, A has velocity 8 m s^{-1} in a direction inclined at 30° to the line of centres of the balls, and B has velocity 4 m s^{-1} in a direction inclined at 60° to the line of centres, as shown in the diagram.



The coefficient of restitution between the balls is $\frac{1}{2}$.

- (a) Find the speed of B immediately after the collision. (9 marks)
- (b) Find the angle between the velocity of B and the line of centres of the balls immediately after the collision. (2 marks)

- 7 A projectile is fired from a point O on the slope of a hill which is inclined at an angle α to the horizontal. The projectile is fired up the hill with velocity U at an angle θ above the hill and first strikes it at a point A . The projectile is modelled as a particle and the hill is modelled as a plane with OA as a line of greatest slope.
- (a) (i) Find, in terms of U , g , α and θ , the time taken by the projectile to travel from O to A . (3 marks)
- (ii) Hence, or otherwise, show that the magnitude of the component of the velocity of the projectile perpendicular to the hill, when it strikes the hill at the point A , is the same as it was initially at O . (3 marks)
- (b) The projectile rebounds and strikes the hill again at a point B . The hill is smooth and the coefficient of restitution between the projectile and the hill is e .



Find the ratio of the time of flight from O to A to the time of flight from A to B . Give your answer in its simplest form. (4 marks)

END OF QUESTIONS

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